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A Convective Model for Light Curves of ZZ Ceti Stars

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Abstract. The erratic behavior of the luminosity in ZZ Ceti light curves may be explained in terms of the sum of several convective fluctuations, where the heat flux is propagated in time by thermal waves. We present simulations of the light curves of GD29-38, GD 358, and HL Tau-76.

1. The Luminosity and Relaxation Time

The ZZ Ceti stars, pulsating hydrogen atmosphere (DA) white dwarfs, exhibit extremely complex light curves. The light curves of ZZ Ceti stars have variations of luminosity with typical intervals between successive pulses in the range from hundreds to thousands of seconds, and in general the "periods" are not constant. Their multi-periodic amplitudes are of the order of 0.001 to 0.5 mag. The oscillations and pulsations (non-radial g modes) are usually attributed to intrinsic temperature variations (Kepler et al. 2000), obtained through modelling of the luminosity variation by spherical harmonics. However, the models proposed for the luminosity variations in these stars are based on semi-analytical approaches for the temperature distribution on the surface. Also convection and diffusion play an important role in the DA atmospheres, as in the thermal evolution. Therefore it is necessary to propose some evolutionary pattern of the complex oscillations observed in the light curves, starting from a formalism based on the usual equations of evolution and stellar structure. A simple model using Mixing Length Theory (MLT) before the thermal relaxation approach, could be good in order to connect the usual stellar evolution formalism with the quasi-analytical approaches (distribution of surface temperatures as a sum of spherical harmonics) in ZZ Ceti light curves. The difference to standard asteroseismological calculations is that the model includes the heat propagation by waves in the WD interior, where the material is degenerate. Furthermore the energy transport equation and the luminosity change if thermal waves are taken into account. The influence and importance of convection and MLT in the study and calculation of atmospheric models for white dwarfs is widely reported. Convection in the higher layers of WD atmospheres occurs before thermal relaxation, therefore the heat transport equation is (see Falcon 2005 and references therein):

$$\frac{dT}{dr} = -\frac{3}{16\pi ac} \cdot \frac{\kappa\rho}{T^3 r^2} \cdot \left(\tau \frac{\partial L}{\partial t} + L\right) \tag{1}$$

where τ is the relaxation time, or the time elapsed until the temperature gradient is switched on after the energy flux appears. Notice that with $\tau \approx 0$ this equation is the "classical" energy transport equation in the stellar interior:

$$L^{(d)} \equiv L(t) = L_{maximun} \cdot exp(-t/\tau_d) \tag{2}$$

However, if the relaxation time is not negligible, i.e. inside WD nucleus thermal waves exist. We now used the Eq. (1) in the MLT convection theory before relaxation (Falcón 2003) in which the luminosity varies quasi-periodicly, thus:

$$L \equiv L^{(d)} \frac{1}{\omega^2+1} exp\left[\frac{t}{4\tau}\left(\omega^2-1\right)\right]\left[\left(5+\omega^2\right) cos\left(\frac{\omega t}{2\tau}\right) - \frac{(3-\omega^2)}{\omega} sin\left(\frac{\omega t}{2\tau}\right)\right] \tag{3}$$

where ω (the observational frequency in light curves) is given by:

$$\omega^2 \equiv \frac{4\tau}{\tau_d} - 1 \tag{4}$$

This equation (3) connects the standard luminosity ($L^{(d)}$) of MLT and the luminosity before thermal relaxation (L) due to heat waves. Note that the luminosity variation is a damped oscillation that depends on: time t, relaxation time τ and thermal adjustment time τ_d. On the other hand, it is possible to calculate the relaxation time from the equation (Falcón & Labrador 2001):

$$\tau = \frac{k}{V^2 C_v} \approx \frac{10^{-3} T^{-2}}{\beta^2} \tag{5}$$

Here the thermal conductivity (k) for degenerate material andfalconP69.tex the specific heat (C_v) by the Chandrasekhar's relationship have been used. Obviously the relaxation time will depend of the value assumed for the thermal waves speed ($V = \beta \cdot c$). Figure 1 shows the relaxation time for various values of temperature and several values of thermal wave speed.

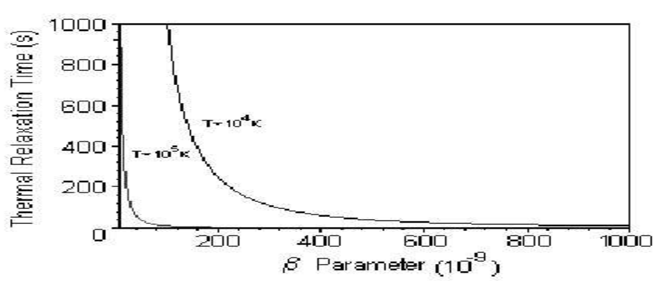

Figure 1. Relaxation time for different temperatures as a function of thermal wave speed

2. The ZZ Ceti Light Curves

The luminosity fluctuation due to the causal propagation of the thermal flux (Eq. 2) suggests an approximate model for the study of the ZZ Ceti stars. Inside the WD the matter is degenerate and the thermal conductivity is dominated by electrons, therefore the use of the Cattaneo Law is justified. Let us assume the existence of statistical fluctuations in density or temperature in some fluid portion in the stellar deep interior. The aleatory movement of certain convective globules along the temperature gradient would create a fluctuation in temperature and luminosity. The true luminosity variation due to the convective flux could have a damped oscillator-like behavior. On time scales comparable to the relaxation time, the total stellar luminosity is due to the contributions of the radiative flux (approximately constant) and even more to the contribution of the convective flux (which is a damped oscillator). According to this model the periods of the luminosity fluctuations would be of the order of τ. Also, because the convective flux is only a fraction of the total thermal flux, the luminosity variations would be damped and small compared to the intrinsic luminosity. The erratic behavior of the ZZ Ceti light curves would be explained in terms of the sum of several convective fluctuations along time.

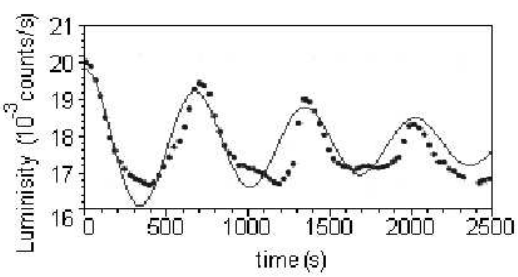

Figure 2. Simulation of a portion of the light curve of G 29-38 for $\tau \approx 1000s$ and $\tau_d \approx 12s$. Observational data by McGraw (Falcón & Labrador 2001)

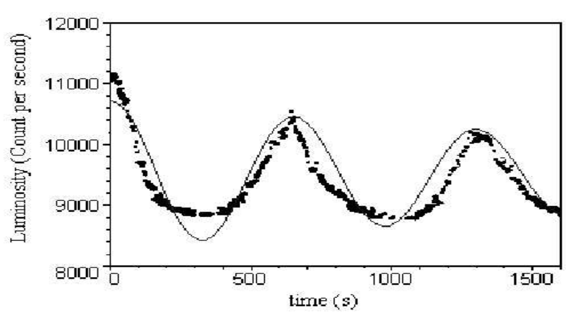

Figure 3. Simulation of a portion of the light curve of HL Tau-76 for $\tau \approx 1336s$ and $\tau \approx 8s$. Observational data by Robinson (1983)

Taking together several convective fluxes, each one due to some specific thermal fluctuation, could reproduce the light curves of some ZZ Ceti stars (see

Figure 4 for the modeling of several portions in the ZZ Ceti light curve of the rapid variable GD358). Here we used the thermal wave speed instead of the sound speed in degenerate material. We can see that the luminosity shows the qualitative behavior of a damped oscillator. The period of the oscillations can be reproduced by appropriate values of τ and τ_d. It can be observed that the theoretical adjustment allows modeling the light curve in pieces; it is consistent with the presented pattern, according to which the fluctuations of the luminosity are caused by each one of the "convective cells" independently.

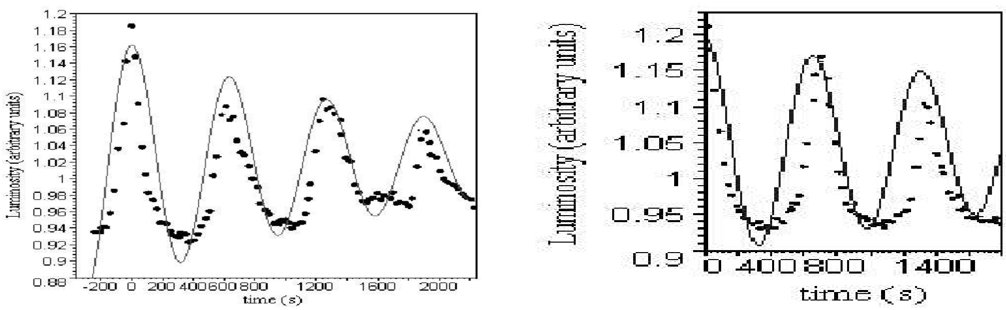

Figure 4. Simulation of a portion of the light curve of GD358 for $\tau \approx 1179s$ and $\tau_d \approx 6s$ (left); and $\tau \approx 1011s$ and $\tau_d \approx 10s$ (right). Observational data by Robinson (1983)

3. Conclusion

The WD quasi periodic variation of luminosity could be a model of the rapid variation of ZZ Ceti stars, because the typical period for the observed oscillations cannot be understood by some acoustic model, but may be owing to a model based on the second sound speed in degenerate material (Falcón & Labrador 2001; heat waves in degenerate material). The presented ideas could be good in order to connect the usual stellar evolution formalism with the quasi-analytical approaches. The erratic behaviour in time of the ZZ Ceti light curves would be explained in terms of the sum of several convective fluctuations.

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